The Assessment of “Understanding”

Study to remember and you will forget.
Study to understand and you will remember.
—Anonymous

I once sat on the dissertation committee of a graduate student in mathematics education who had examined whether advanced graduate students in math and science education could explain the logic underlying a popular procedure for extracting square roots by hand. Few could explain why the procedure worked. Intrigued by the results, she decided to investigate whether they could explain the logic underlying long division. To her surprise, most in her sample could not. All of the students were adept at division, but few understood why the procedure worked.

In a series of studies at Johns Hopkins University, researchers found that first year physics students could unerringly solve fairly sophisticated problems in classical physics involving moving bodies, but many did not understand the implications of their answers for the behavior of objects in the real world. For example, many could not draw the proper trajectories of objects cut from a swinging pendulum that their equations implied.

What then does it mean to “understand” something—a concept, a scientific principle, an extended rhetorical argument, a procedure or algorithm? What questions might classroom teachers ask of their students, the answers to which would allow a strong inference that the students “understood”? Every educator from kindergarten through graduate and professional school must grapple almost daily with this fundamental question. Do my students really “get it”? Do they genuinely understand the principle I was trying to get across at a level deeper than mere regurgitation? Rather than confront the problem head on, some teachers, perhaps in frustration, sidestep it. Rather than assign projects or construct examinations that probe students’ deep understanding, they require only that students apply the learned procedures to problems highly similar to those discussed in class. Other teachers with the inclination, time and wherewithal often resort to essay tests that invite their students to probe more deeply, but as often as not their students decline the invitation and stay on the surface.

I have thought about issues surrounding the measurement of understanding on and off for years, but have not systematically followed the literature on the topic. On a lark, I conducted three separate Google searches and obtained the following results:

- “nature of understanding” 41,600 hits
- “measurement of understanding” 66,000 hits
- “assessment of understanding” 34,000 hits

Even with the addition of “classroom” to the search, the number of hits exceeded 9,000 for each search. The listings covered the spectrum—from suggestions to elementary school teachers on how to detect “bugs” in children’s understanding of addition and subtraction, to discussions of laboratory studies of brain activity during problem solving, to abstruse philosophical discussions in hermeneutics and epistemology. Clearly, this approach was taking me everywhere, which is to say, nowhere.

Fully aware that I am ignoring much that has been learned, I decided instead to draw upon personal experience—some 30 years in the classroom—to come up with a list of criteria that classroom teachers might use to assess understanding. The list is undoubtedly incomplete, but it is my hope that it will encourage teachers to not only think more carefully about how understanding might be assessed, but also—and perhaps more importantly—encourage them to think more creatively about the kinds of activities they assign their classes. These activities should stimulate students to study for understanding, rather than for mere regurgitation at test time.

The student who understands a principle, rule, procedure or concept should be able to do the following tasks (these are presented in no particular order and their actual difficulties are an empirical question):

Construct problems that illustrate the concept, principle, rule or procedure in question.
As the two anecdotes above illustrate, students may know how to use a procedure or solve specific textbook problems in a domain, but may still not fully understand the principle involved. A more stringent test of understanding would be that they can construct problems themselves that illustrate the principle. In addition to revealing much to instructors about the nature of students' understanding, problem construction by students can be a powerful learning experience in its own right, for it requires the student to think carefully about such things as problem constraints and data sufficiency.

Identify and, if possible, correct a flawed application of a principle or procedure.
This is basically a check on conceptual and procedural knowledge. If a student truly understands a concept, principle or procedure, she should be able to recognize when it is faithfully and properly applied and when it is not. In the latter case, she should be able to explain and correct the misapplication.

Distinguish between instances and non-instances of a principle; or stated somewhat differently, recognize and explain "problem isomorphs," that is, problems that differ in their context or surface features, but are illustrations of the same underlying principle.
In a famous and highly cited study by Michelene Chi and her colleagues at the Learning Research and Development Center, novice physics students and professors of physics were each presented with problems typically found in college physics texts and asked to sort or categorized them into groups that "go together" in some sense. They were then asked to explain the basis for their categorization. The basic finding (since replicated in many different disciplines) was that the novice physics students tended to sort problems on the basis of their surface features (e.g., pulley problems, work problems), whereas the experts tended to sort problems on the basis of their "deep structure," the underlying physical laws that they illustrated (e.g., Newton's third law of motion, the second law of thermodynamics). This profoundly revealing finding is usually discussed in the context of expert-novice comparisons and in studies of how proficiency develops, but it is also a powerful illustration of deep understanding.

Explain a principle or concept to a naïve audience.
One of the most difficult questions on an examination I took in graduate school was the following: "How would you explain factor analysis to your mother?" That I remember this question over 30 years later is strong testimony to the effect it had on me. I struggled mightily with it. But the question forced me to think about the underlying meaning of factor analysis in ways that had not occurred to me before.

Mathematics educator and researcher, Liping Ma, in her classic exposition *Knowing and Teaching Elementary Mathematics* (Lawrence Erlbaum, 1999), describes the difficulty some fifth and sixth grade teachers in the United States encounter in explaining fundamental mathematical concepts to their charges. Many of the teachers in her sample, for example, confused division by 1/2 with division by two. The teachers could see on a verbal level that the two were different but they could neither explain the difference nor the numerical implications of that difference. It follows that they could not devise simple story problems and other exercises for fifth and sixth graders that would demonstrate the difference.

To be sure, students may well understand a principle, procedure or concept without being able to do all of the above. But a student who can do none of the above almost certainly does not understand, and students who can perform all of the above tasks flawlessly almost certainly do understand.

One point appears certain: relying solely on the problems at the end of each chapter in text books, many of which have been written by harried and stressed-out graduate students, will not assure that our students understand the concepts we wish to teach them. The extended essay has been the solution of choice for many instructors whose teaching load and class size permit such a luxury. But less labor intensive ways of assessing understanding are sorely needed.